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Use of Discontinuous Grids for Solving Helmholtz Equation in Complex Models

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SUMMARY

In this paper, we consider 2D Helmholtz equation with complex shift and assume it to be solved using domain decomposition technique where finite-difference method on fine and coarce meshes applied in the upper and lower parts of the model respectively. We proved that the use of different grids leads to nonsymmetrical perturbation of the original operator, thus causes the irreducible error in the solution – artificial reflections from the boundaries of the subdomains, even though the residual tends to zero exponentially with respect to the number of iteration.

Introduction

Numerical solution of Helmholtz equation is one of the main problems in modern seismic modelling as it is an essential part of full waveform inversion and reverse time migration procedures. However, complexity of the medium may require advanced techniques to be used. In particular the upper part of the model typically has a complex structure with low velocity zones, irregularity of interfaces and free surface profile. Meanwhile, the major part of the model can be relatively simple with no sharp interfaces and no extremely low velocities. This makes it reasonable to use different discretizations in the upper and lower parts of the model with further implementation of the domain decomposition technique (DD) to combine the two statements. Collino et al.(1998) and Gander et al.(2007) demonstrated that one of the main parameter governing the convergence of the DD iterative process in the quality of the transmission conditions, used at the interfaces between the subdomains. However, study of the DD is typically applied to the differential statement without taking into account difference in approximation of the differential operator by the discrete ones..

In this paper, we consider 2D Helmholtz equation and assume it to be solved using domain decomposition technique where finite-difference method on fine and coarce meshes applied in the upper and lower parts of the model respectively. We proved that the use of different grids leads to nonsymmetrical perturbation of the original operator, thus causes the irreducible error in the solution even though the residual tends to zero exponentially with respect to the number of iteration.

Method

Domain decomposition technique, applied to the Helmholtz equation with complex shift β can be written as follows:

$$\Delta u_L^N + \frac{\omega^2}{c^2(x,z)}(1+i\beta)u_L^N = f(x,z)$$
$$\frac{du_L^N}{dz} - \frac{i\omega}{c(x,z)}\sqrt{1+i\beta}u_L^N = 0\Big|_{z=0}$$
$$r_L\left[u_L^N\right] = r_R\left[u_R^{N-1}\right]_{z=z_L}$$
$$z \in \left[0, z_L\right],$$

$$\Delta u_R^N + \frac{\omega^2}{c^2(x,z)}(1+i\beta)u_R^N = f(x,z)$$
$$l_R\left[u_R^N\right] = l_L\left[u_L^{N-1}\right]_{z=z_R}$$
$$\frac{du_R^N}{dz} + \frac{i\omega}{c(x,z)}\sqrt{1+i\beta}u_R^N = 0\Big|_{z=H}$$
$$z \in \left[z_R, H\right],$$

where $0 < z_1 < z_R \le z_L < z_2 < H$, $\beta < 0$, l_R, l_L, r_R, r_L are the perturbed transmission operators defined at the interfaces $z = z_L$ and $z = z_R$ (Figure 2). Subscripts L and R denote the subdomain in which the solution is defined. The superscript correspond number of iteration.

To investigate the error caused by the perturbation of the original operator it is convenient to introduce a skewness function F – characterizing the difference in the approximation errors in upper and lower subdomains. It means that if the Helmholtz operator is approximated by the same numerical method with the same discretization etc. in both subdomains the skewness will be zero, however in our case, where different grids and different transmission conditions are used in the two subdomains F is not zero. As the result the norm of the solution error can be estimated as follows: $\|\varepsilon_{N+2}\| = \|S\varepsilon_N + (A+I)F\|$, which converges to $\|\varepsilon_{2k}\| \le \left\|\left(\frac{1}{1-\lambda}S + I\right)(A+I)\right\| \cdot \|F\|$, where λ is eigenvalue of an operator S mapping the solution from iteration N to N+2. These formulae mean that in general case presence of nonsymmetric perturbations in the approximations lead to irreducible error in the solution – artificial reflections from the nonphysical interfaces due to domain decomposition.

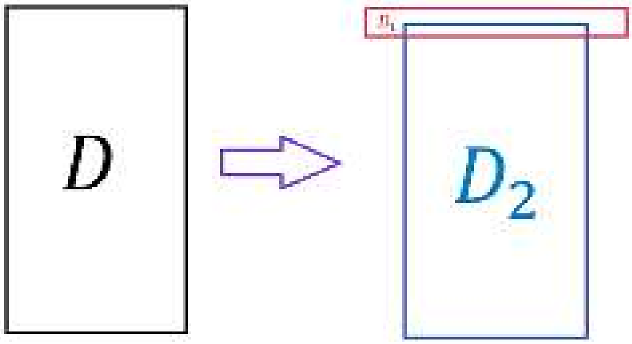

Figure 1 *Decomposition of computational domain into two subdomains with an overlap.*

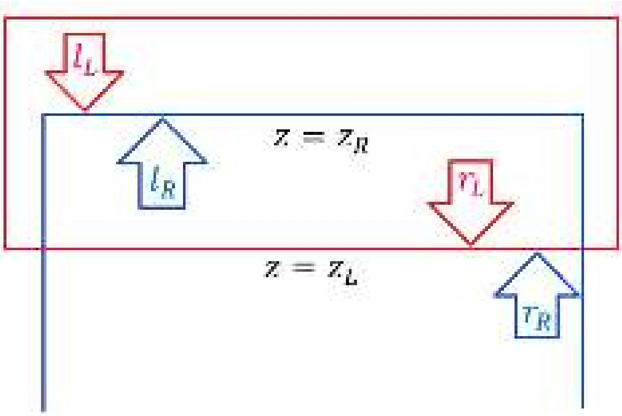

Figure 2 *Transmission conditions between the subdomains.*

Examples

To illustrate the effect of the use of different discretizations of the Helmholtz equation in adjoin subdomains we considered the problem stated inside the a domain $z \in [0,2500]$, $x \in [0,2000]$, with the velocity model provided in fig. 3 (Gullfaks velocity model). The frequency was $\omega = 15 \cdot 2\pi$. The source location is $z = 150$, $x = 1000$.
The computational domain was split into 2 subdomains with an overlap: small upper part - $z \in [0,250]$ and the main part - $z \in [201,2500]$ (Figure 3). Helmholtz equation was approximate by the second order finite-difference scheme with grid steps equal to $h_x = h_z = 2.5$ in the upper subdomain and $h_x = h_z = 5.0$ used in the lower part of the model. As the result the skewness of the operator perturbation was nonzero causing irreducible error in the solution. The iterative process stops when residual (difference between transmission conditions) drops down to 10^{-5} . For the considered model the iteration process converged in 11 iterations. The solution and the error are provided in fig.4. It is clearly seen that the error is mainly affected by the presence of the interface between the grids; i.e. by skewness of the operator perturbation. (Figure 4).

The following 3 series of experiments are for different damping factor value $\beta = -0.05, -0.1, \ldots, -0.45, -0.5$ which is used to improve convergence of the Krylov-type methods where DD is used as a preconditionner as in Neklydov et al.(2010) . Three different statement were considered:

- fine grids used in both subdomains
- coarse grids used in both subdomains
- fine grid in the upper subdomain and coarse grid in the lower subdomain.

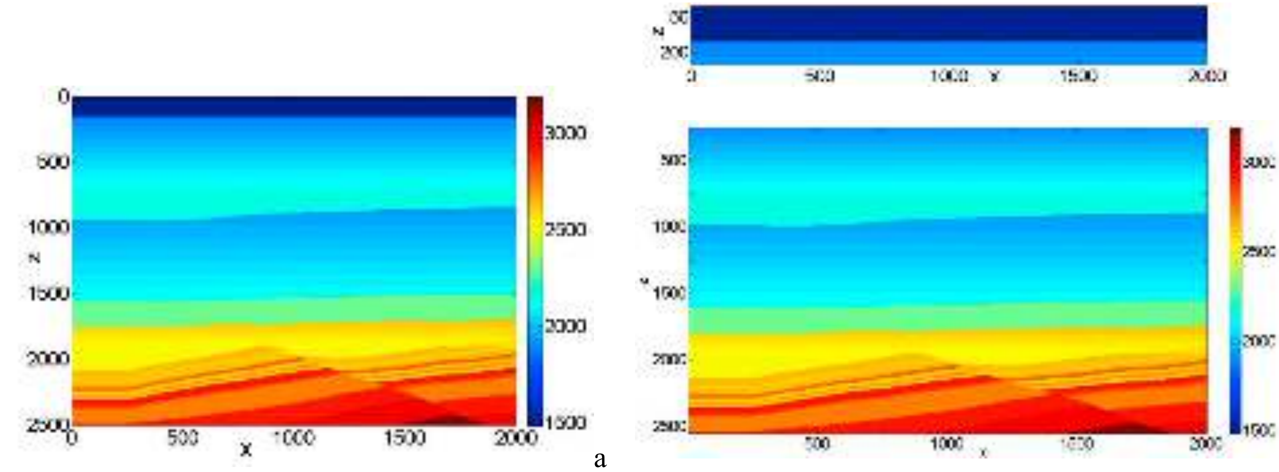

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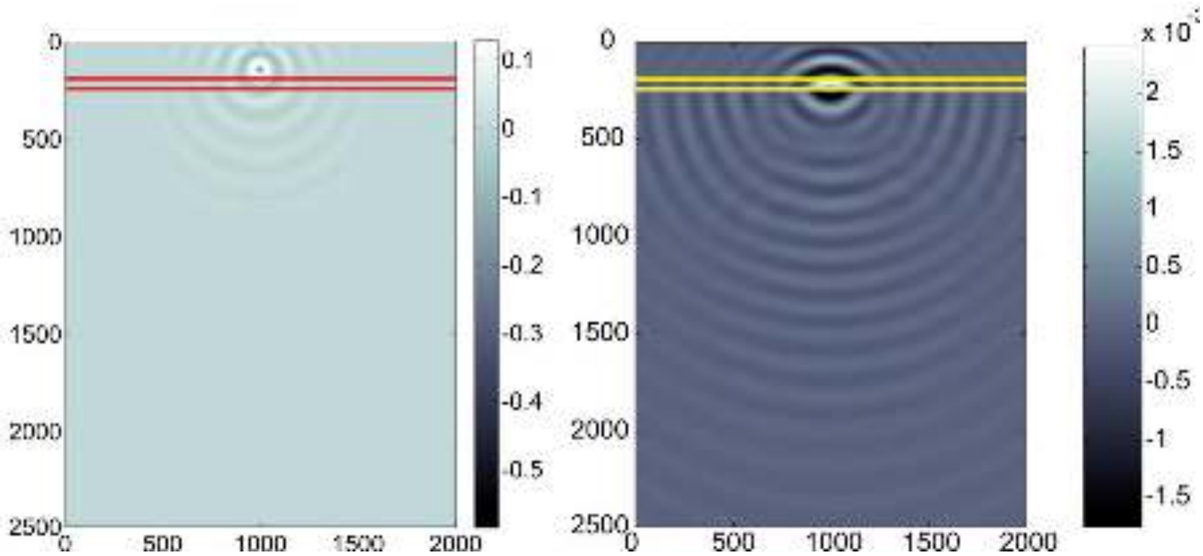
Figure 3 *Gullfaks velocity model (Vp).*

Figure 4 *Solution in computed without DD (left), difference between solution computed with and without DD (right) – absolute error.*



According to the numerical results increase in damping factor improves the convergence rate of the DD method (fig.5) and also decrease the artificial error caused by the use of discontinuous grids (fig.6). Indeed introduction of the damping leads to artificial attenuation of the error, which is widely used in classical DD formulation for elliptic problems.

Conclusions

An effect of the combination of different grids with domain decomposition method for solving Helmholtz equation was studied in the paper. It was proved that the use of different numerical methods or different discretizations in the joint subdomains leads to nonsymmetrical perturbation of the original operators and the solution of the domain decomposition problem suffers from irreducible error – artificial reflections from the boundaries of the subdomains.

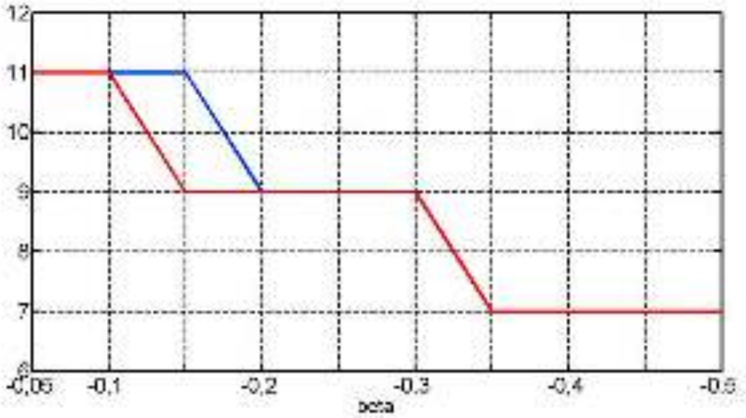


***Figure 5** Dependence of the DD iteration number on β. Blue line corresponds to different grids realization and red line represents the experiment with coarse grid discretization.*

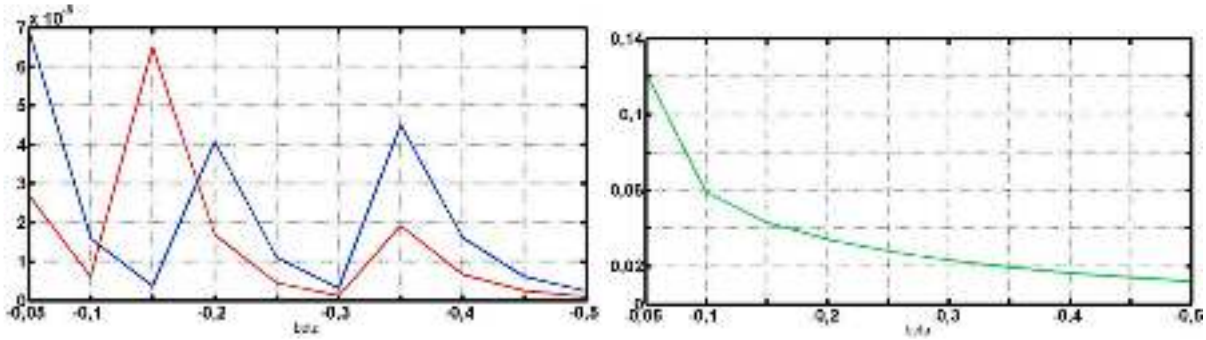


***Figure 6** Error of the DD solution with respect to β. Blue line represents the fine grid solution, red line corresponds to the coarse grid experiments, and green line corresponds to the experiment with different grids.*

Acknowledgements

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